Math 10A with Professor Stankova
Worksheet, Discussion #40; Monday, 12/4/2017
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Problems

1. Let $f(x) = \sqrt{\sin(x)}$. Find $f'(x)$.

2. Find the equation for the tangent line to $(x^2 + y^2 - 2x)^2 = x^2 + y^2$ at the point $(0, 1)$.

3. Find $\lim_{x\to\infty} \frac{x+e^{-x}}{2x+1}$.

4. Find $\lim_{x\to\infty} \frac{\sqrt{x}}{\ln x}$.

5. Find $\lim_{x\to 0} \frac{1-\cos(x)}{\sec(x)}$.

6. Find $\lim_{x\to 0^+} x^{\sin x}$.

7. Graph $y = x^4 - 2x^2$.

8. You are standing at the shore and there is a ball 15 meters along the shore and 6 meters out. You run at a rate of 6.4 m/s and swim at rate of $0.91 m/s$. In order to get to the ball, you run some distance along the shore and then swim in a diagonal path to the ball. How long should you run on the shore to get to the ball the fastest?

9. A cone shaped funnel is 25 cm in height and has a radius 10 at the top. Water is flowing out the bottom at a rate of 2.5 cm^3/sec. What is the rate at which the height h of the water is dropping when $h = 15$.

10. Find the roots of $g(x) = x^3 + x - 5$ (explain how many and how you would find them).

True/False

11. True False The vertical line test tests whether a curve in the plane is the graph of a function.

12. True False Integration and differentiation are inverse processes linked by the Fundamental Theorem of Calculus.

13. True False Every one-to-one function has an inverse.

14. True False Every exponential function has a doubling time.

15. True False The doubling time of $y = 3^x$ is a period of this function.

16. True False The log-log plot turns a power function into a linear function, but we need a semi-log plot to turn an exponential function into a linear one.

17. True False Limit laws are at the bottom of the Calculus structure.

18. True False We can always plug in $x = c$ to find the limit $\lim_{x\to c} f(x)$ except when the function is not continuous at $x = c$.

19. True False If Limit Laws do not apply from the get-go when attempting to find $\lim_{x\to c} f(x)$, then the limit does NOT exist.

20. True False Problem-solving techniques (PSTs) are often used to find limits $\lim_{x\to c} f(x)$ when we have a hard time plugging in $x = c$.

21. True False The definition of limit for functions is responsible for the proof of the Limit Laws.

22. True False Horizontal asymptotes can NEVER have any points in common with the graph of a function.

23. True False Vertical asymptotes CAN have common points with the graph of a function.

24. True False While a limit $\lim_{x\to c} f(x)$ does not care what happens exactly at $x = c$ because the limit is concerned only with the behavior of $f(x)$ nearby $x = c$, continuity does care about both and wants them to coincide.

25. True False There are at least two pieces of the Calculus structure that are responsible for proving DL+, one of which is the definition of continuity.

26. True False A composition of two continuous functions, as long as it is well-defined, is always continuous.

27. True False The tangent slope of a function is the limit of infinitely many secant slopes.

28. True False In the definition of the derivative $f'(x_0), x_0$ varies while $h \to 0$.

29. True False When calculating the derivative of a function by the derivative definition, we can never first plug in $h = 0$ because we will inevitably get $\frac{0}{0}$; instead, we must first simplify until we cancel h from top and bottom of the fraction, and only then we can apply LLs or basic limits of well-known functions.

30. True False There is a geometric and an algebraic interpretations of the derivative, and the two do not always address the same concept for some weird functions.

31. True False To prove that $(e^x)' = e^x$, we can reduce the problem to calculating the limit $\lim_{h\to 0} \frac{e^h-1}{h}$, which in turn is precisely the definition of the derivative $(e^x)'|_{x=1}$.

32. True False After we define what derivatives are, we try to get away from the definition of derivative as fast as possible by discovering and proving DLs because it is cumbersome to use the definition all the time; yet, for some complicated functions that do not comform to any of our DLs we are bound to go down again to the basics, start from scratch, and use the definition of derivative.

33. True False Defining e as the only real number a for which $\lim_{h\to 0} \frac{a^h-1}{h} = 1$ is vastly inconvenient for calculating the value of e, while definition e as the limit $e = \lim_{n\to\infty}(1+1/n)^n$ is quite practical as it gives us the opportunity to calculate e with whatever precision we like; yet, the two choices are equivalent and yield the same value of e.

34. True False Reciprocal functions have inverse derivatives.

35. True False If a function is not differentiable at $x = c$, then it cannot be continuous there either.

36. True False The Chain Rule can be used to provide shortcut formulas for derivatives of reciprocals of functions and, more generally, for derivatives of powers of functions.

37. True False Implicit differentiation is typically used when we are trying to compute the derivative of a function given by a complex formula $y = f(x)$.

38. True False Turning $y \mapsto y(x)$ is a suggested (but not absolutely necessary) step when doing implicit differentiation because otherwise it is hard to apply CR and in the process we may incorrectly omit some of the derivatives $y'(x)$.

39. True False The second derivative test for concavity is NOT a bullet-proof test because in none of the possible 4 cases can we make any definitive conclusions about the function.

40. True False If the first derivative changes its sign, we are absolutely sure that the original function has a local extremum at x_0 too.

41. True False Using the graph of $f'(x)$, we can sketch many graphs of the possible original functions $f(x)$.

42. True False When x_0 is not in the domain of $f(x)$, we cannot automatically assume that $f(x)$ has a vertical asymptote there; instead, we need to find out what $\lim_{x\to x_0^+} f(x)$ and $\lim_{x\to x_0^-} f(x)$ are and those could be different or non-existent.

43. True False As done in class, when constructing the table to study the graph of $f(x)$, we should always include the zeros of $f(x)$ as special inputs as they will never unnecessarily clutter this table, unless such an x_0 is also a zero of the first and/or second derivative of $f(x)$, in which case we should never include such an x_0 in the table.

44. True False The logarithmic derivative of $f(x)$ explains why the log-log plot of $f(x)$ may be a better representation of the function's rate of change relative to its size.

45. True False The logarithmic derivative of $f(x)$ can also be used as a sleek approach to calculating the derivative of $f(x)$.

46. True False Working with related rates of change is similar to implicit differentiation in that we are finding the derivative of some function without explicitly finding a formula for this function.

47. True False When a differentiable function on a closed interval has only one critical point, then the function is bound to have a global maximum or minimum there.

48. True False When a rational function $f(x)$ has an asymptote $y = c$ for $x \to +\infty$, then $f(x)$ also has the same asymptote $y = c$ for $x \to -\infty$; however, this is not necessarily true for a general function $f(x)$.

49. True False We often rescale $e^{-\frac{x^2}{2}}$ to $\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ because a function should be more complicated in order to be useful.

50. True False Similar triangles are a geometric tool that can help us solve some Calculus problems.

51. True False When plugging values for x into the Taylor polynomials of trig. functions, we have to make sure that these values are in degrees, not radians, because it is simpler to work in degrees.

52. True False All of the derivatives of the nth Taylor polynomial $T_n(x)$ at $x = a$ of a function $f(x)$ and the function itself are equal at $x = a$.

53. True False We know that the $\sin(x)$ function is an odd function because it is a strange function and, in addition, all powers of x appearing in its Taylor polynomials at $a = 0$ are of odd degrees.

54. True False Quadratic approximations are, in general, better than linear approximations on larger intervals around the center $x = a$.

55. True False Calculators use Taylor expansions to compute values of functions that would otherwise be hard or impossible to compute by hand.

56. True False Factorials appear in the denominators of terms in Taylor polynomials due to the fact that $(x^n)^{(n)} = n!$ for all $n = 0, 1, 2, \ldots$.

57. True False L'Hopital's Rule (LH) is a dangerous tool in the hands of people who do not verify its conditions before applying it.

58. True False If the conditions for L'Hopital's Rule (LH) are satisfied, then we can calculate the given limit by applying LH once or several times.

59. True False When the first derivative $f'(a) \neq 0$, we can use it to determine if the quadratic approximation of $f(x)$ at $x = a$ tends to be an overestimate or an underestimate of $f(x)$.

60. True False We expand $\ln(1 + x)$ at $a = 0$ instead of $\ln x$ at $a = 0$ because we want to practice finding Taylor polynomials on more complicated functions.

61. True False Newton's method is a more sophisticated version of using Taylor Polynomials, since when it does works, it works faster.

62. True False Newton's method utilizes repeatedly the linear approximations (or tangent lines) for $f(x)$, but taken at various points on the graph.

63. True False Newton's method can fail if we chose the initial x_1 too far from the intended root, if in the process some derivative $f'(x_n) = 0$ or does not exist, or for no good reason.

64. True False The formula for Newton's method involves the same ratio of a function $f(x)$ and its derivative as we saw in the "Black Cloud" example, which also equals the logarithmic derivative of $f(x)$.

65. True False $\sqrt{3}$ can be approximated by using Taylor Polynomials and by Newton's method; however, different functions are needed in each approach.

66. True False There are no formulas for solving degree 3 polynomial equations, and hence we must use Taylor polynomials to approximate the roots of these polynomials.

67. True False L'Hospital's Rule (LH) can be used for finding limits in cases of product and exponential indeterminancies, but some preparation work needs to be done to rewrite the problem into a quotient indeterminacy before applying LH.

68. True False If for a function defined and twice differentiable on $\mathbb{R}$ we find out that its first derivative has some root r and its second derivative is everywhere negative, then we can conclude that the function has one local (= global) minimum.

69. True False Newton's method is useful for approximating critical points of a function.

70. True False We can rewrite an exponential indeterminacy as a product indeterminacy and then use CR and PR to find the derivative of the original function.